THE EFFECTS OF CHANGES IN FORMATION PERMEABILITY AROUND THE WELL BORE ON THE PRODUCTION BEHAVIOR OF A WELL

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by

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Thesis
P273

FOREWORD

The author wishes to express his appreciation to Professor [illegible] L. Priest, Head of the Petroleum Engineering Department, University of Pittsburgh, for his helpful suggestions and comments in the preparation of this paper; and to the U. S. Naval Post Graduate School, Monterey, California, through whose sponsorship this paper was made possible.

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I. INTRODUCTION

The process of recovering oil from a subsurface reservoir and the rate at which it may be produced depends on the physical characteristics of the reservoir rock and the properties of the fluids contained therein. Reservoir engineering, the science of the production of oil from a subterranean formation, has been developed within the last thirty years. During this period extensive laboratory research has been conducted to ascertain the physical composition and properties of the fluids contained in the reservoir and the mathematical relationships that describe their flow through the reservoir formation at the prevailing temperatures and pressures. Many analytical studies have been presented to describe the flow of these reservoir fluids both quantitatively and qualitatively.

It has been found that for a uniformly permeable formation which is completely saturated with a wetting liquid, the equations of flow are relatively simple. For a formation of varying permeability with compressible fluid flow the mathematical analysis becomes more complicated. Finally, for a formation which contains three phases (water, oil and gas) the mathematical analysis becomes extremely complex. Resort to approximation in the mathematical analysis must be made for practical applications in three-phase fluid flow.

In 1856 Darcy discovered that the rate of linear flow of a fluid through a permeable medium completely saturated with the fluid was directly proportional to the pressure difference between the two points of flow and inversely proportional to the fluid viscosity. This constant of proportionality was termed "permeability" and was defined as "a

measure of the capacity of the porous medium to permit flow of a fluid through its interconnecting pores. The unit of permeability is the darcy and is defined as the rate of flow in milliliters per second of a fluid having a viscosity of one centipoise, through a cross section of one square centimeter under a pressure gradient of one atmosphere per centimeter, and conditions of viscous flow."[1] This concept is the fundamental relationship upon which the equations of fluid flow are based.

In 1859, the famous Drake oil well was completed at Titusville, Pennsylvania, which marked the first time a well was specifically drilled for oil in the United States. The race for oil was on and numerous wells were drilled with varying degrees of success.

During the next few decades emphasis was placed on exploration techniques with little regard for improvement in completion methods, production processes, and controls. As a result, wells were damaged beyond repair and millions of barrels of oil were irretrievably lost.

As more and more wells were brought in and the demand for oil grew, more interest was displayed in the factors which influenced the flow of oil through the reservoir to the well. Laboratory investigations were undertaken and mathematical analyses were made to determine the physical characteristics of the oil and the equations of flow which would describe the behavior of the oil in the reservoir during the production process.

In 1937 M. Muskat published an exhaustive study on the mathematical analysis of fluid flow entitled "The Flow of Homogeneous Fluids

---

[1] References are listed in the Bibliography.

[illegible] PERMEABILITY OF THE MEDIUM

Numerous books, articles, and technical papers have been written discussing the flow of fluids through a reservoir during the production process. Equations have been developed and graphical interpretations have been made [illegible] through a reservoir to a producing well under varying conditions of [illegible] economics of recoverable reserves of petroleum in [illegible] rates which may be expected to be reached during the life of the well. [illegible] the oil [illegible] saturation) the effective permeability is synonymous with the formation permeability. However, if gas, which is dissolved in the oil, comes out of solution during production, the oil saturation will decrease and the effect is to reduce the relative permeability of the formation to oil which will result in proportionately less oil in the flowing stream for a given pressure gradient. This paper will be restricted to the flow of a single homogeneous fluid.

Formation permeability depends on the cross-sectional area and tortuosity of the paths in the formation through which the fluid flows. These properties depend on the grain size distribution, type of packing, and degree of cementation. Formations which contain clay may have a

usually reduced permeability near the well bore due to [illegible] water from the drilling mud penetrating into the formation, thereby causing the clay to swell and plug the pores of the formation. Reduced permeability can also be caused by [illegible] or improper handling procedures during well completion. Conversely, the permeability around the well bore may be substantially increased over that of the remainder of the formation by acidizing or fracturing which effectively increases the cross-sectional area of the paths of flow.

Studies of the productive performance of a well are usually based on the pressure distribution from the well bore to the outer boundary of the drainage area. It can be shown that for a six inch well with uniform permeability, and with normal well spacing of 1320 feet, 50% of the pressure drop between the outer boundary of the drainage area and the well occurs within 20 feet of the well bore. Because of this, any change of the formation permeability within a short distance of the well can substantially change the productivity index.

The technique most commonly used to determine the degree of change of permeability around the well bore compared to the permeability of the formation away from the well bore is the pressure build-up curve. From this curve can be derived the reservoir pressure, the formation permeability away from the well bore, and the degree of change of formation permeability around the well bore.

The purpose of this thesis is to develop additional simplified concepts and procedures to further the understanding of the effects of permeability change around the well bore on the productivity of the well. An analytical review of the basic equations of radial flow of homogeneous fluids through a permeable medium, and the pressure build-up curve will be presented.

III. REVIEW OF THE THEORY OF RADIAL FLOW

[illegible]

Note: The following development follows that given in Muskat's "The Flow of Homogeneous Fluids Through Porous Media."

A. Steady State Flow of a Single, Non-compressible Fluid Through a Uniformly Permeable Medium[2]

The basic equation of flow is the Laplace equation*

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) = 0 \qquad (1)$$

The general solution of this type of equation is:

$$p = C_1 \ln r + C_2 \qquad (2)$$

where: C_1 and C_2 are the constants of integration

The boundary conditions are:

$p = p_w$ when $r = r_w$

$p = p_e$ when $r = r_e$

Substituting these values in equation (2) the pressure equation for equation (1) is:

$$p - p_w = \frac{p_e - p_w}{\ln r_e/r_w}\ln r/r_w \qquad (3)$$

The velocity potential $= \frac{k}{\mu}p$

Therefore the radial velocity $(v_r) = \frac{k}{\mu}\frac{\partial p}{\partial r}$

---

*Symbols and nomenclature are listed in Table 1 of the Appendix.

$$[illegible] = p(r_e) - [illegible] \frac{(p_e - p_w)}{\ln r_e/r_w}$$

Where: [illegible]   Where: A = surface radial area at any radius r = 2πrh

Substituting the value of A_r:

$$Q = \frac{2\pi kh\,(p_e - p_w)}{\mu \ln r_e/r_w} \quad \text{[illegible]}$$

To convert from reservoir cc. to stock tank cc. [illegible] we must divide by the formation volume factor (β):

$$Q = \frac{2\pi kh\,(p_e - p_w)}{\mu\beta \ln r_e/r_w} \quad \text{(milliliters of S.T.O./sec)} \qquad (4)$$

If the practical system of units is used in equation (4) we can rewrite equation (4) as:

$$Q = \frac{0.00708\,kh\,(p_e - p_w)}{\mu\beta \ln r_e/r_w} \qquad (5)$$

By substituting the value of $(p_e - p_w)$ from equation (5) into equation (3) [illegible]

$$p = p_w + \frac{Q\mu\beta}{[illegible]} \ln \frac{r}{r_w} \qquad (6)$$

The pressure distribution from the well bore outward to the outer boundary of the drainage area as given by equation (6) is plotted as [illegible] of the Appendix.

[illegible]

[illegible]

[illegible]

[illegible]

[illegible] of different potentials [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible] area #1 and area #2, which is at a distance r_o from the center of the well. The following boundary conditions [illegible] $P_{[illegible]}$ [illegible]

(a) [illegible]

(b) [illegible]
[illegible]

[illegible]

$$P_1 = P_2 \qquad \text{when } r = r_o$$

$$k_1 \frac{\partial P_1}{\partial r} = k_2 \frac{\partial P_2}{\partial r}$$

and

$$P_1 = P_w \qquad \text{when } r = r_w$$

$$P_2 = P_e \qquad \text{when } r = r_e$$

[illegible] equations [illegible]

Zone 1

$$r_w \le r \le r_o \qquad P_{(r_w - r_o)} = P_w + \frac{P_e - P_w}{\ln r_o/r_w + \frac{K_1}{K_2} \ln r_e/r_o} \ln r/r_w \qquad (11)$$

Zone 2

$$r_o \le r \le r_e \qquad P_{(r_o - r_e)} = P_e + \frac{\frac{K_1}{K_2}(P_e - P_w)}{\ln r_o/r_w + \frac{K_1}{K_2} \ln \frac{r_e}{r_o}} \ln r/r_e \qquad (12)$$

Since there must be continuity of flow from the [illegible] boundary to the well bore through zones #1 and #2:

$$Q = \frac{2\pi k h}{\mu} r \frac{\partial P_{(r_w - r_o)}}{\partial r} \qquad (13)$$

[illegible]

[illegible] (1) [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible] for various degrees of compression [illegible] in figures [illegible], and [illegible] of the appendix.

The [illegible] of [illegible] which is [illegible] from a [illegible] areas of different permeability can be [illegible] of a [illegible] from a [illegible] of uniform permeability [illegible]

[illegible]

[illegible] and rearranging [illegible]

[illegible]

[illegible] Fluid Flow of a Single [illegible] through a Permeable [illegible] of Different Permeability

[illegible] outer boundary of the reservoir [illegible] rate of a compressible fluid [illegible]

[illegible]

[illegible] the practical application [illegible] equation (1) [illegible]

[illegible] permeability change in the [illegible]

[illegible]

[illegible]

[illegible]

following form:

[illegible]

[illegible] identical to the equation proposed by Horner.

IV. [illegible]

[illegible]

$$[illegible]$$ ([illegible])

[illegible]

$$[illegible]$$ ([illegible])

[illegible]

1. [illegible] at a constant rate from the center of an [illegible] reservoir with a constant pressure at the external boundary.

2. [illegible] present at the [illegible].

3. The compressibility and absolute viscosity of the [illegible] constant over the range of temperature and pressure variation encountered.

4. The well is [illegible] face and there is no [illegible].

5. The formation permeability is homogeneous in the [illegible].

These [illegible] are only approximated under actual field conditions. A [illegible] of the [illegible] between the [illegible] of the effect of these variances on the [illegible] of equation [illegible] follows.

[illegible]

[illegible] for the pre- [illegible] and the [illegible]

[illegible]

[illegible]

[illegible] could be [illegible] producing from a formation [illegible]

[illegible] the permeability of the forma- [illegible]

[illegible] the expression for the [illegible] will be:

[illegible]

[illegible]

[illegible]

[illegible] evaluating the Damage Factor, Completion Factor, and the Condition Factor. It has been shown that although the numerical values of these factors are different for a given problem, [illegible] differences [illegible] the pressure drawdown theoretically obtained for a given rate of production from a well producing from a uniformly permeable formation, and [illegible]

Since both the Completion Factor and the Damage Factor can be determined graphically from a pressure build-up curve, for practical application either of these two factors is recommended over the Condition Factor for determining the relative change of permeability around the well bore.

[illegible] recommended for comparison. A [illegible]

[illegible]

[illegible] Pressure Build-up Curve

[illegible]

[illegible] rate for a minimum period [illegible] it then run into the [illegible] is shut-in. Readings of [illegible] as the pressure build up is [illegible] a theoretical interpretation [illegible] so that the [illegible], production, and well [illegible]

[illegible] set of data, presented by the author, [illegible]

	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible] μ	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

BEFORE ACIDIZATION		AFTER ACIDIZATION	
Well Pressure	$\frac{\Delta t}{T + \Delta t}$	Well Pressure	$\frac{\Delta t}{T + \Delta t}$
[illegible]	[illegible]	3750	2×10^{-7}
[illegible]	[illegible]	[illegible]	$10^{[illegible]}$
[illegible]	[illegible]	[illegible]	$10^{[illegible]}$
[illegible]	[illegible]	3865	10^{-4}
[illegible]	[illegible]	3920	10^{-3}
[illegible]	[illegible]	3960	5×10^{-3}
[illegible]	[illegible]	4000	2.5×10^{-2}

[illegible]

1. Plot "well pressure" versus $\frac{\Delta t}{T + \Delta t}$ on semi-logarithmic paper. The semi-logarithmic plot is included in the Appendix as Figure 7.

2. Extrapolate the straight line portion of the build-up curves to $\frac{\Delta t}{T + \Delta t} = 10^{0}$. Read the average reservoir pressure (p_e) = 4210 psi.

3. Determine the formation permeability away from the well bore. Slope (Z) = 60 psi/log cycle.

$$k = \frac{[illegible]\, \mu \beta}{Z\, h} = [illegible] = [illegible]$$

4. Determine the value of $\frac{\Delta t}{T + \Delta t}$ when $\Delta t = 0$

Before Acidizing

$$\frac{\Delta t}{T + \Delta t} = \frac{f \mu c\, r_w^{2}}{[illegible]} = [illegible]$$

$$= [illegible]$$

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible] less the value [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

r_e = 5,280 [illegible]

[illegible]

[illegible]

[illegible] the [illegible] into the [illegible]

[illegible]

[illegible]

[illegible] following procedures are recommended:

1. [illegible] is the [illegible] of the well if there is a change of permeability of the [illegible]. If the permeability of the [illegible], the production rate will [illegible] life of the well.

2. [illegible] production so that the various factors [illegible] may be determined.

3. If the permeability of the [illegible] the well core is [illegible]

[illegible]

The [illegible] economically recoverable reserves of a [illegible] many variable factors [illegible] the reservoir, the composition and physical [illegible] produced fluids (oil, water and gas), [illegible], the production rate, and the type of displacement [illegible] however, in order to demonstrate [illegible] a decrease in formation permeability, with a re- [illegible].

[illegible]

[illegible] well. It is assumed that the oil is above the bubble point (no gas liberated from the oil) and that only oil is produced until the well turns to water.

The equation for the cumulative production (R) may be simply stated as:

[illegible] where: S = number of days of production until well turns to water.

Since Q is directly proportional to the permeability, if the permeability is reduced, the production rate will be reduced in the same proportion. If we assume the sweep efficiency of the water drive to be the same at the reduced rate of production as it is at the initial rate, the cumulative production will be the same although the production time will be increased in the inverse ratio of the permeability change.

Depletion Gas Drive

In a depletion gas drive the expulsive force is provided by the expansion of gas initially dissolved in the oil and liberated as the reservoir pressure drops during the production. It is known that as production progresses, the production rate will drop off in the form of a "decline curve." Some of the main causes of these curves are: (a) the permeability of the formation and (b) the pore volume. It appears doubtful that the [illegible] would hold without a mathematical basis, only [illegible] of predicting production from any type well during depletion.

The following formula is based on a form of loss-ratio decline curve as defined by the equations below:

$$Q = Q_o (1-c)^t \qquad (42)$$

$$R = D (Q_o - Q) \qquad (43)$$

$$D = \frac{-1}{\ln (1-c)} \qquad (44)$$

The values of the terms will be based on the problem presented in Section IV, C. A decline (c) of 10% per year and an economical production rate limit of 5 bbls./day will be assumed.

Initially, the well had a formation permeability of 10.5 md away from the well bore, a permeability of approximately half as great

[illegible] barrels per day [illegible]

Solving for [illegible]

[illegible]

The [illegible] is:

[illegible]

[illegible] (11) and solving for the total number of [illegible]

[illegible] years

[illegible] under the initial conditions, 1,200,000 barrels of oil [illegible]

If the well [illegible] so that the permeability around the [illegible] increased to 10.[illegible], for the same drawdown the initial production rate would be increased to:

Q = [illegible] barrels per [illegible]

If the decline rate is increased in the [illegible] permeability change:

[illegible]

[illegible]

[illegible]

The [illegible]ative production would then be:

$$R = 1970\ ([illegible]-5) = 1,[illegible],000 \text{ barrels}$$

The total number of production years would be:

$$[illegible] = [illegible] \text{ years}$$

[illegible]

[illegible] the effect of reduction of [illegible] economically recoverable [illegible] [illegible] production process, each [illegible] investigated and analyzed. However, [illegible] [illegible] increased. This will be [illegible] yearly income derived from the pro-

[illegible]

[illegible]

[illegible] porous medium and the manner [illegible] fects the flow. Special emphasis [illegible] formation permeability around [illegible] well.

[illegible] fluid through a formation con- [illegible] permeability can be mathe- [illegible] use of equivalent [illegible] to show the effects on the [illegible]

[illegible] the solution [illegible] better than [illegible]

[illegible]

[illegible] theoretically obtained for a given rate [illegible] flow.

A [illegible] of the [illegible] the pressure drawdown in the formation more clearly, than the [illegible] available in the literature.

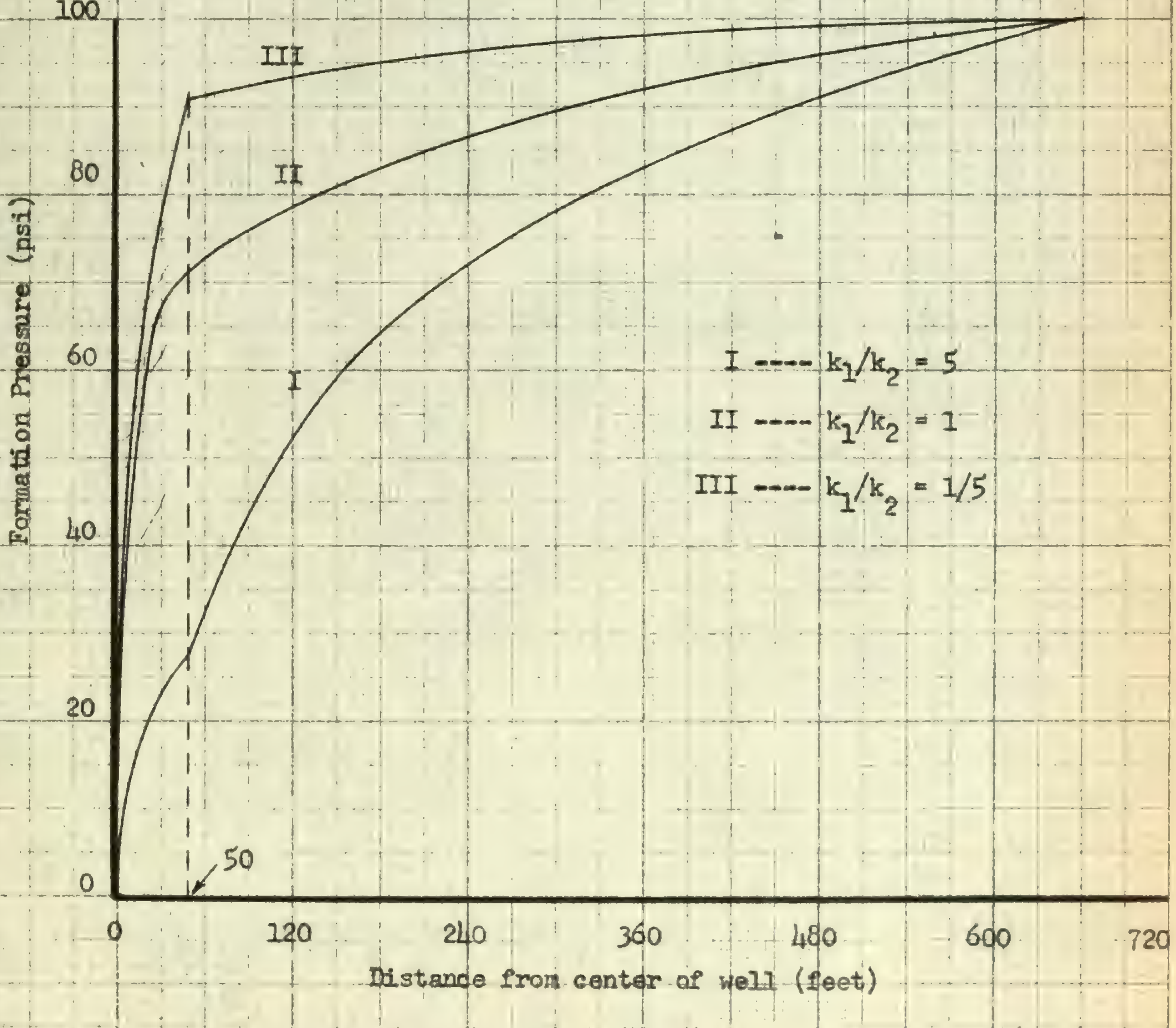
The theoretical aspects of the pressure build-up curve were [illegible] a new procedure was developed for determining the average [illegible] formation permeability is directly proportional to the slope of a straight line [illegible] at infinite time after close-in, and [illegible] the well at the time it is closed in. [illegible] value obtained agreed within few per cent with the value of the average formation permeability calculated by means of equations developed for non-compressible fluid flow.

A practical problem was presented to illustrate the use of the theories of fluid flow and the pressure build-up curve to predict the behavior of the well during the production life of the well.

Finally, two types of displacement mechanisms were briefly discussed, water drive and dissolved gas drive, and a sample problem for each type was presented to illustrate the effect of reduced production

PRESSURE DISTRIBUTION FROM THE WELL TO THE OUTER BOUNDARY OF THE DRAINAGE AREA

(SOURCE: Muskat's "Physical Principles of Oil Production," page 244)



k_1 = Permeability of damaged section

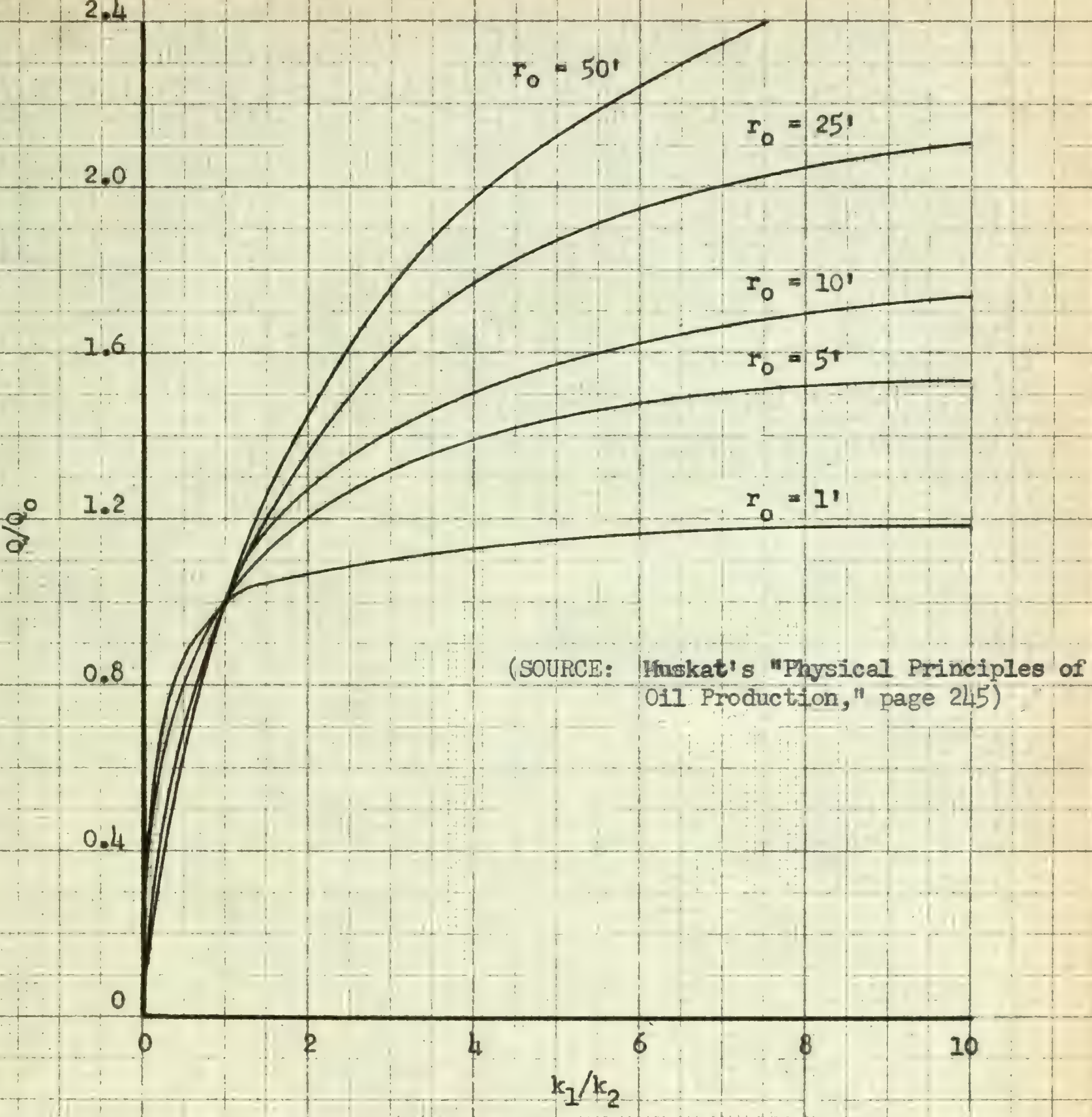
k_2 = Permeability of undamaged section

r_o = 50 feet $r_w = \frac{1}{4}$ foot

r_e = 660 feet $p_w = 0$ p_e = 100 psi

FIGURE 1

PRODUCTIVITY INCREASE FOR DAMAGED OR ACIDIZED SECTIONS

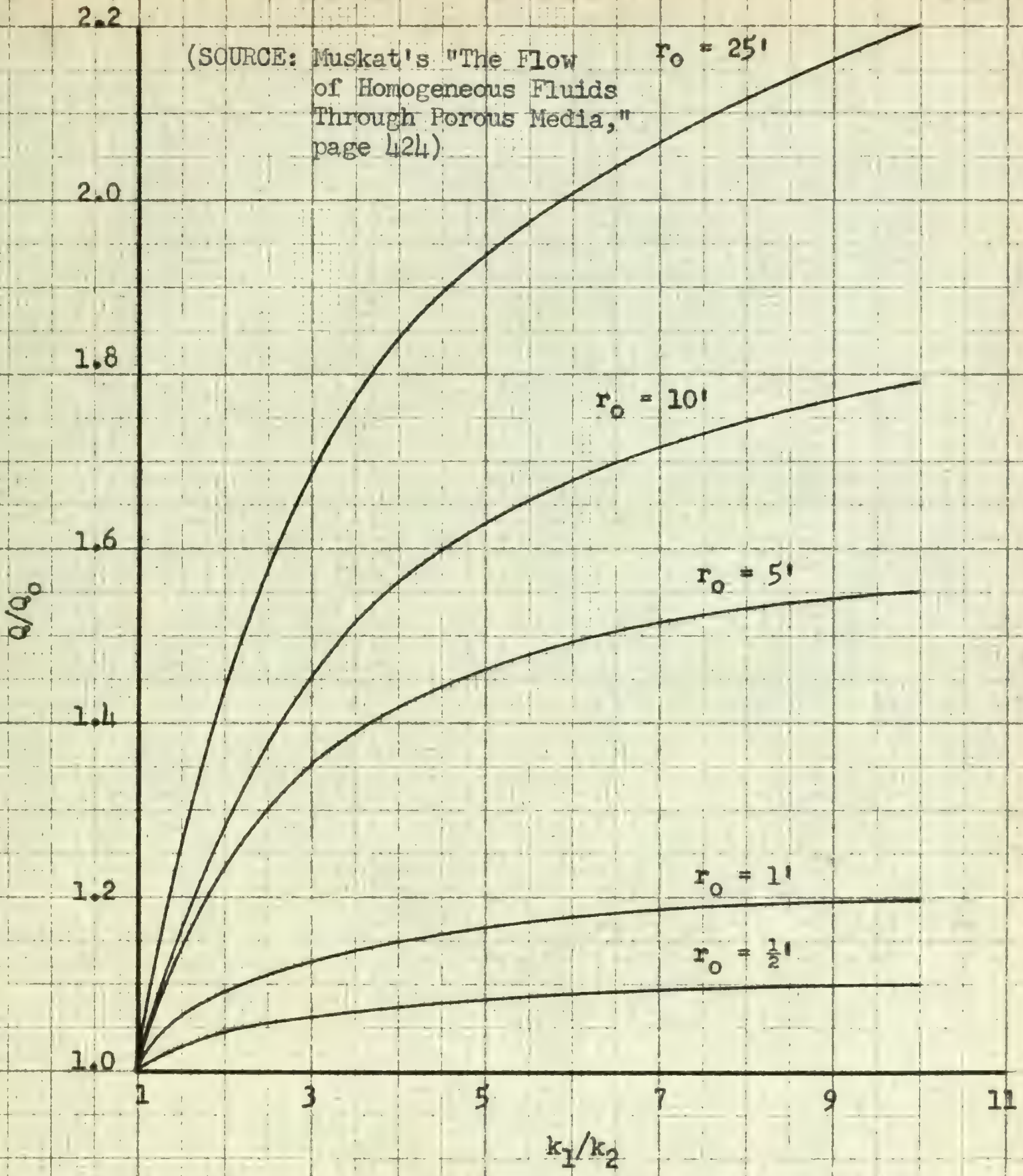


(SOURCE: Muskat's "Physical Principles of Oil Production," page 245)

Q = Production rate with damaged or acidized sections
Q_o = Production rate for undamaged section
k_1 = Permeability of damaged (or acidized) section
k_2 = Permeability of undamaged section
r_w = $\frac{1}{4}$ foot
r_e = 660 feet

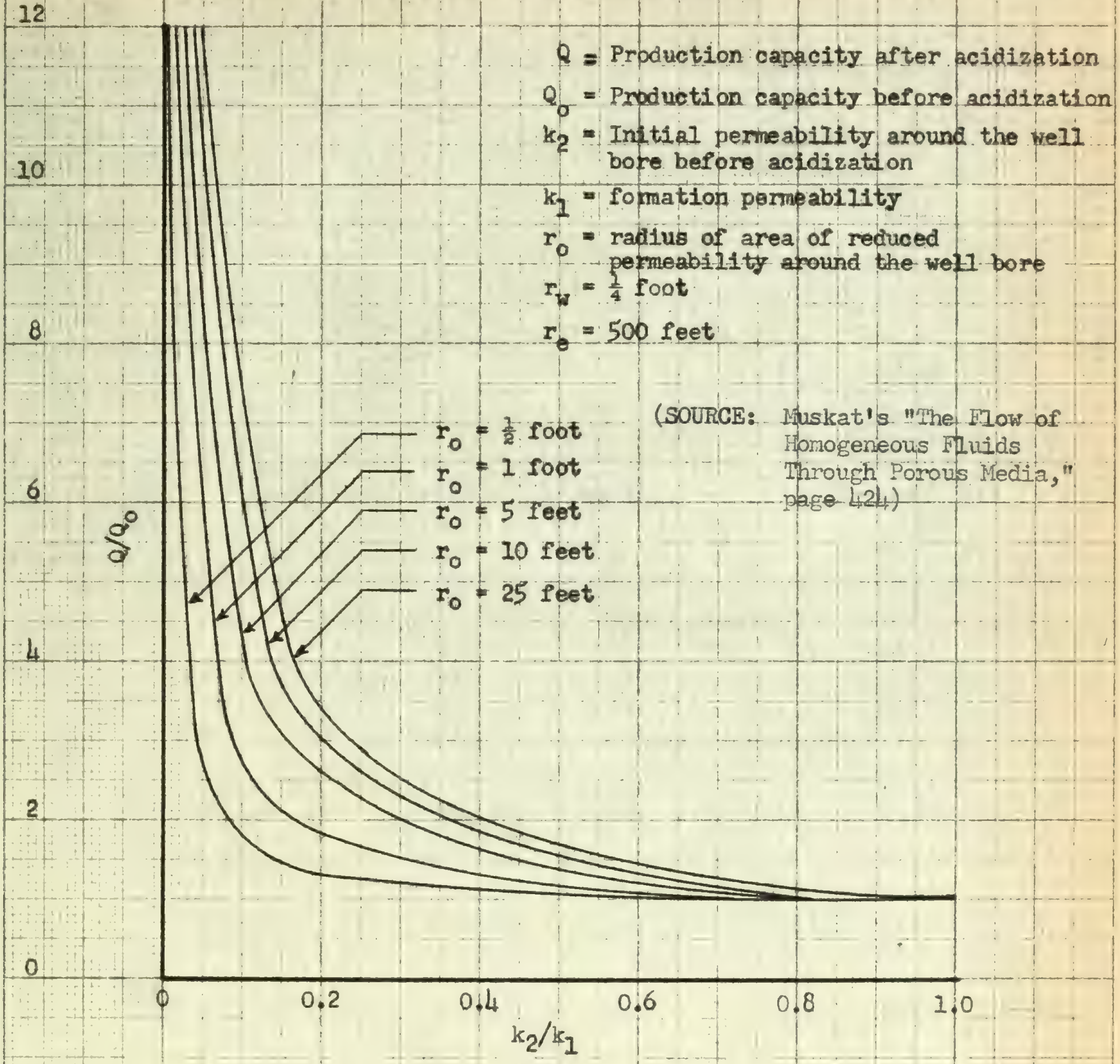
FIGURE 2

PRODUCTIVITY INCREASE DUE TO ACIDIZATION IF THE INITIAL PERMEABILITY IS EVERYWHERE UNIFORM


(SOURCE: Muskat's "The Flow of Homogeneous Fluids Through Porous Media," page 424)

Q = Production capacity after acidization
Q_o = Production capacity before acidization
k_1 = Permeability of acidized zone of radius (r_o)
k_2 = Permeability of acidized zone before acidization
r_w = ¼ foot
r_e = 500 feet

FIGURE 3



PRODUCTIVITY INCREASE DUE TO ACIDIZATION IF INITIAL PERMEABILITY AROUND THE WELL BORE IS RAISED TO FORMATION PERMEABILITY

Q = Production capacity after acidization

Q_o = Production capacity before acidization

k_2 = Initial permeability around the well bore before acidization

k_1 = formation permeability

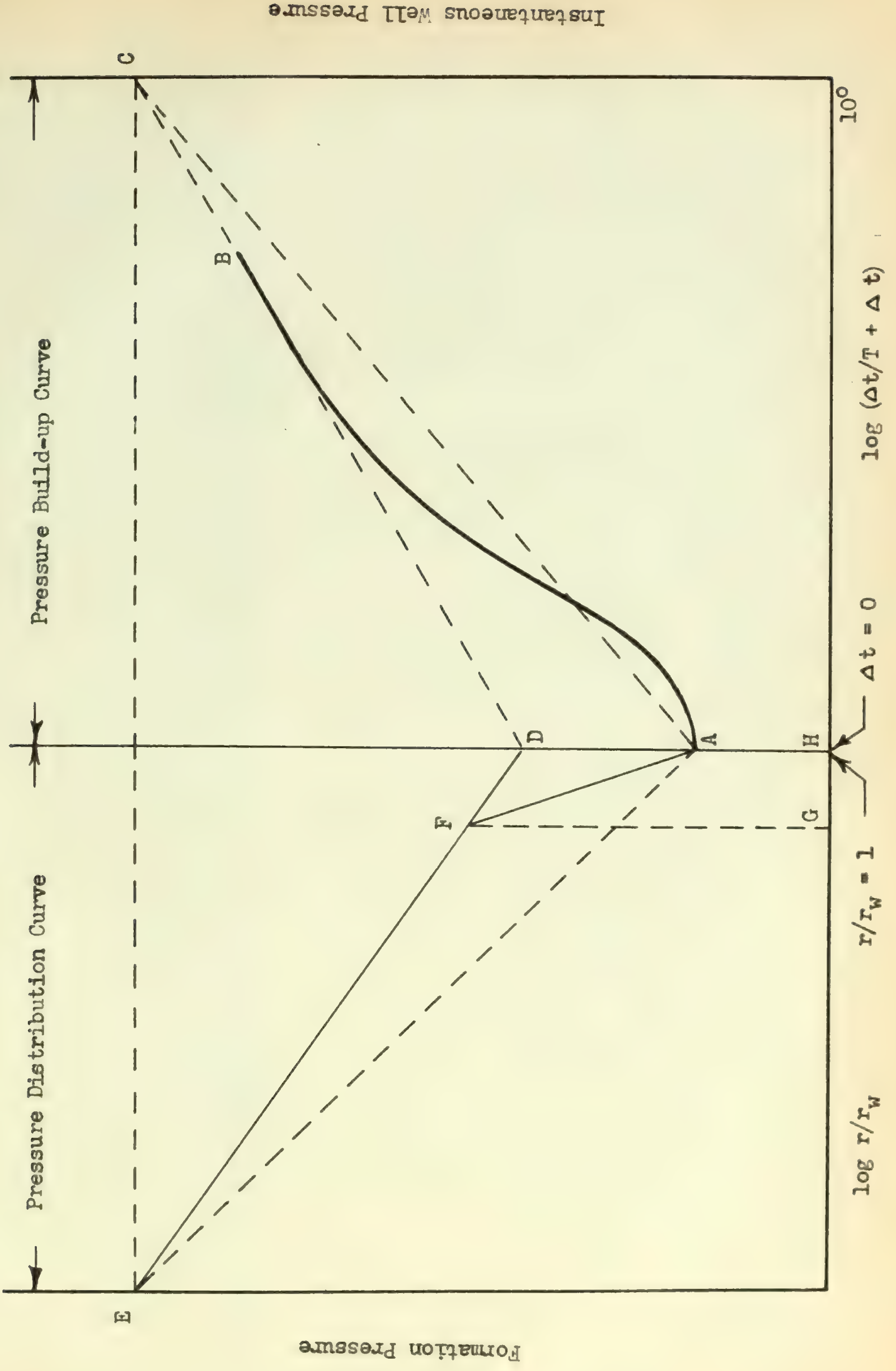
r_o = radius of area of reduced permeability around the well bore

$r_w = \frac{1}{4}$ foot

r_e = 500 feet

(SOURCE: Muskat's "The Flow of Homogeneous Fluids Through Porous Media," page 424)

FIGURE 4


Instantaneous Well Pressure
Formation Pressure
Pressure Build-up Curve
Pressure Distribution Curve
A
B
C
D
E
F
G
H
10^0
log (Δt/T + Δt)
Δt = 0
$r/r_w = 1$
log r/r_w

DETERMINATION OF AVERAGE FORMATION PERMEABILITY

FIGURE 6

NO. 340 L5-10 DIETZGEN GRAPH PAPER
SEMI-LOGARITHMIC
5 CYCLES X 10 DIVISIONS PER INCH

EUGENE DIETZGEN CO.
MADE IN U. S. A.

NO. 340 L5-10 DIETZGEN GRAPH PAPER
SEMI-LOGARITHMIC
5 CYCLES X 10 DIVISIONS PER INCH

EUGENE DIETZGEN CO.
MADE IN U. S. A.

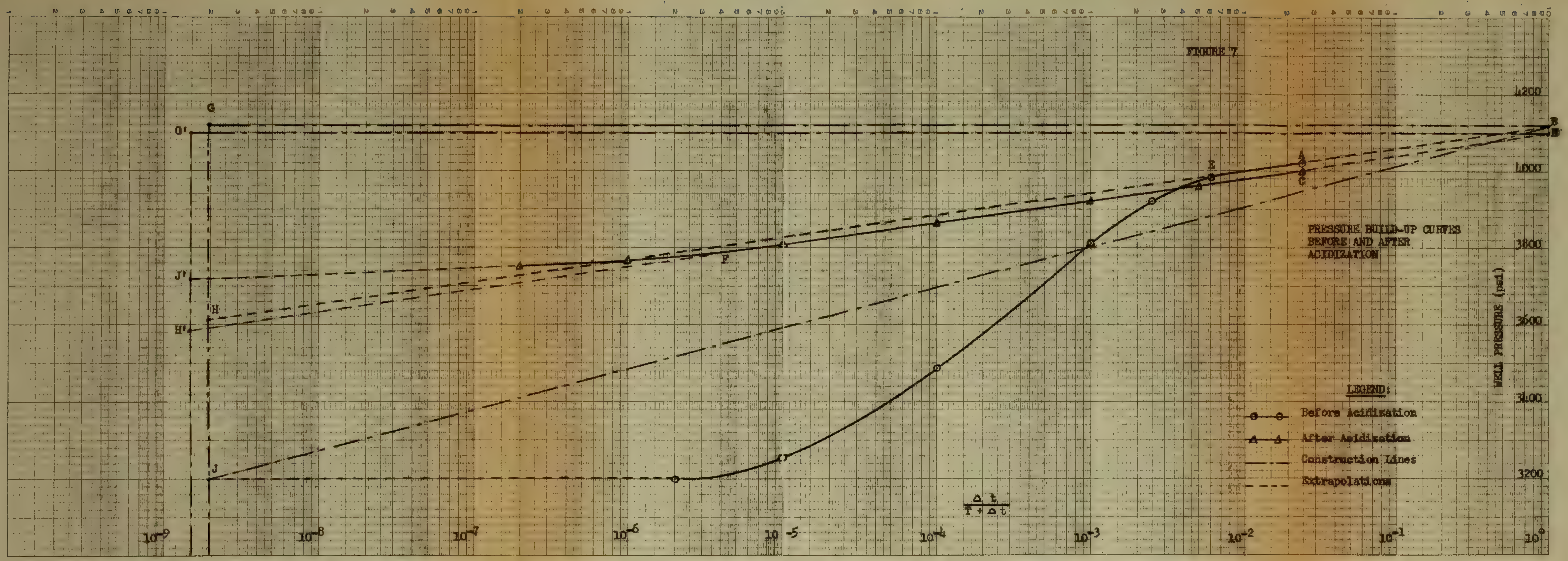


FIGURE 7

PRESSURE BUILD-UP CURVES BEFORE AND AFTER ACIDIZATION

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The effects of changes in formation permeability around the well bore ...

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The effects of changes in formation permeability around the well bore on the production behavior of a well.

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